Idzik and Zapart *Fixed Point Theory and Applications* 2012, **2012**:11
http://www.fixedpointtheoryandapplications.com/content/2012/1/11

Fixed Point Theory and Applications
a SpringerOpen Journal

RESEARCH **Open Access**

Local properties of simplicial complexes

Adam Idzik[1,2] and Anna Zapart[3*]

* Correspondence: A.Zapart@mini.pw.edu.pl
[3]Faculty of Mathematics and Information Science, Warsaw University of Technology, Warsaw, Poland
Full list of author information is available at the end of the article

Abstract

Retractable, collapsable, and recursively contractible complexes are examined in this article. Two leader election algorithms are presented. The Nowakowski and Rival theorem on the fixed edge property in an infinite tree for simplicial maps is extended to a class of infinite complexes.

Keywords: collapsable $\leq n$-complex, perfect elimination scheme, retractable $\leq n$-complex

1 Introduction

By N we denote the set of natural numbers. Let V be a nonempty set and $I_n = \{0, \ldots, n\}$ ($n \in N$). $\mathbb{P}(V)$ is the family of all nonempty subsets of V and $\mathbb{P}_n(V)$ (resp., $\mathbb{P}_{\leq n}(V)$) is the family of all subsets of V of cardinality $n+1$ (resp., at most $n + 1$), $n \in N$. An element of $P_n(V)$ is called an n-simplex (or an n-dimensional simplex) defined on the set V and a nonempty family $\mathbb{K}_n \subset \mathbb{P}_n(V)$ of n-simplices defined on V is called an n-complex defined on the set V (or an n-dimensional complex).

A complex generated by an n-simplex S is the complex $\mathbb{K}_{\leq n}(S) = \{V : V \subset S, V \neq \emptyset\}$. We denote $\overline{S} := \mathbb{K}_{\leq n}(S)$.

Generally, a complex $\mathbb{K}_{\leq n}$ (or an $\leq n$-complex $\mathbb{K}$) defined on the set V is the union of some complexes generated by i-simplices, $i \in I_n$, i.e., $\mathbb{K}_{\leq n} \subset \mathbb{P}_{\leq n}(V)$, and for any simplex $S \in \mathbb{K}_{\leq n}$, $\mathbb{K}_{\leq n}(S) \subset \mathbb{K}_{\leq n}$. A 0-simplex is called a vertex. We denote by $V(\mathbb{K})$ the set of all vertices of $\mathbb{K}$.

Two vertices of a complex are adjacent, if they both belong to a simplex belonging to this complex.

Simplices of a complex are adjacent, if they have a common vertex.

A star at a vertex p (in an $\leq n$-complex $\mathbb{K}$) is the $\leq n$-complex $st_{\mathbb{K}}(p) = \{\overline{S} : p \in S \in \mathbb{K}\}$; the vertex p is also called a center of a star.

Let $S \in \mathbb{K}_{\leq n}$ be an i-simplex of a complex $\mathbb{K}_{\leq n}$. Then the i-simplex S is a single i-simplex (of $\mathbb{K}_{\leq n}$) if there exists exactly one $(i + 1)$-simplex $T \in \mathbb{K}_{\leq n}$ such that $S \subset T$ ($i \in I_{n-1}$); compare Definition 2.60 [1] of a free face.

A complex $\mathbb{L}_{\leq m} \subset \mathbb{K}_{\leq n}(m \leq n)$ is obtained by an elementary collapse of a $\leq n$-complex $\mathbb{K}_{\leq n}$ if there is a single i-simplex $S \subset T \in \mathbb{K}_{\leq n}$ and $\mathbb{L}_{\leq m} = \mathbb{K}_{\leq n} \setminus \{S, T\}$, where T is the unique $(i + 1)$-simplex containing S ($i < n$); see [2] and compare the definition of d-collapsing in [3].

The definition above is more precise than the definition of an elementary collapse of a complex [4]. It is similar to an elementary collapse of a cube (see Definition 2.64 in [1]).

We say that an $\leq n$-complex is $\mathbb{K}_{\leq n}$ collapsable to an $\leq m$-complex $\mathbb{K}_{\leq m}(\mathbb{K}_{\leq m} \subset \mathbb{K}_{\leq n}, m \leq n)$ if and only if there are subcomplexes $\mathbb{L}^{k+1}, \mathbb{L}^{k}, \ldots, \mathbb{L}^{0}$, such that $\mathbb{L}^{i}$ is obtained by an elementary collapse of $\mathbb{L}^{i+1}(i \in I_k)$, $\mathbb{L}^{k+1} = \mathbb{K}_{\leq n}$ and $\mathbb{L}^{0} = \mathbb{K}_{\leq m}$, for some $k \in N$.

An $\leq n$-complex $\mathbb{K}_{\leq n}$ is collapsable, if it is collapsable to one vertex.

For a simplex $S = \{p_0, \ldots, p_n\} \in \mathbb{K}_{\leq n}$ we denote its boundary by $\partial S := \{\{p_0, \ldots, \hat{p}_i, \ldots, p_n\} : i \in I_n\} \subset \mathbb{K}_{\leq n}$, where $\hat{p}_i$ means that the vertex p_i is omitted.

Notice that for an $(n + 1)$-simplex S, ∂S is an n-complex consisting of all n-subsimplices of S.

Let u, v be adjacent vertices of a complex $\mathbb{K}_{\leq n}$ and let V be the set of its vertices. We call a map $r : V \to V\backslash\{u\}$ defined by $r(u) = v$ and $r(x) = x$ for $x \in V\backslash\{u\}$, a retraction if:

(i) u and v do not belong to the boundary $\partial S \subset \mathbb{K}_{\leq n}$ of some simplex $S \notin \mathbb{K}_{\leq n}$,

(ii) the complex $\mathbb{K}'_{\leq n}$ defined on vertices $V\backslash\{u\}$ with simplices $S \in \mathbb{K}_{\leq n}$, such that $u \notin S$ or $S = S'\backslash\{u\} \cup \{v\}$ for some $S' \in \mathbb{K}_{\leq n}$ and $S' \ni u$, is a subcomplex of $\mathbb{K}_{\leq n}$.

A complex $\mathbb{K}'_{\leq n}$ which can be obtained from a complex $\mathbb{K}_{\leq n}$ by a finite sequence of retractions is called a retract of the complex $\mathbb{K}_{\leq n}$.

A complex $\mathbb{K}_{\leq n}$ is retractable if it can be reduced, by a sequence of retractions, to one vertex.

A union of complexes $\mathbb{K}_i(i \in I_n)$ is the complex $\mathbb{L} = \bigcup_{i \in I_n} \mathbb{K}_i$ with vertices $V(\mathbb{L}) = \bigcup_{i \in I_n} V(\mathbb{K}_i)$.

Analogously, an intersection of complexes $\mathbb{K}_i(i \in I_n)$ is the complex $\mathbb{L} = \bigcap_{i \in I_n} \mathbb{K}_i$ with vertices $V(\mathbb{L}) = \bigcap_{i \in I_n} V(\mathbb{K}_i)$.

A graph G is a nonempty set $V(G)$, whose elements are called vertices, and a set $\mathbb{E}(G) \subset \mathbb{P}_{\leq 1}(V(G))$ of elements of unordered pairs of the set $V(G)$ called edges. In case an unordered pair consists of a vertex, it is called a loop.

For convenience we identify the graph with the respective complex $\mathbb{K}_{\leq 1}$.

2 Retractable complexes

Observe that an $\leq n$-complex $\mathbb{K}$ is precisely defined by its vertices $V(\mathbb{K}) := \bigcup_{S \in \mathbb{K}} S$ and its maximal simplices $\max \mathbb{K} := \{S : S \in \mathbb{K};$ there is no T such that $S \subset T \in \mathbb{K}$ and $S \neq T\}$.

For complexes $\mathbb{K}_{\leq n}$ and $\mathbb{L}_{\leq m}$ a map $f : V(\mathbb{K}_{\leq n}) \to V(\mathbb{L}_{\leq m})$ is called simplicial if every simplex of $\mathbb{K}_{\leq n}$ is mapped onto some simplex of $\mathbb{L}_{\leq m}$.

We say that a complex $\mathbb{K}$, with the vertices $V(\mathbb{K}) = \bigcup_{S \in \mathbb{K}} S$ has the fixed simplex property if for every simplicial map $f : V(\mathbb{K}) \to V(\mathbb{K})$ there exists a simplex $S \in \mathbb{K}$ which is mapped onto itself, i.e., $f(S) = S$.

For retractable $\leq n$-complexes the fixed simplex property is valid:

Theorem 2.1 ([5], Theorem 2.3). If an $\leq n$-complex is retractable, then it has the fixed simplex property.

The above result implies the Hell and Nešetřil theorem: any endomorphism of a dismantlable graph fixes some clique [6].

Notice that retractable $\leq n$-complexes may have only one vertex which begins a sequence of retraction (see Figure 1).

In the example of Figure 1, the only possible retraction maps u to the vertex v. Thus, we can not obtain any vertex as a retract (the vertex u is not possible to obtain in this case).

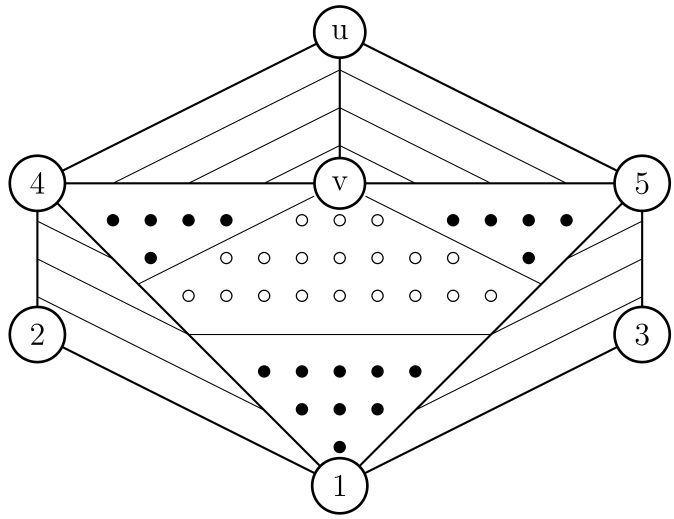

Figure 1 A retractable complex $\mathbb{K}_{\leq 2}$ = $\{\{123\}$, {124}, {135}, {23*v*}, {24*v*}, {35*v*}, {4*uv*}, {5*uv*}, {12}, . . . } with only vertex *u* beginning retractions.

Fact 2.2. For the retraction of a vertex u to a vertex v the vertices adjacent to the vertex u are also adjacent to the vertex v. Thus a retraction is a simplicial map.

For a retractable complex we can define a local algorithm to obtain a vertex of this complex.

Algorithm 2.3 [reducing a retractable complex to a vertex]:

Input: any retractable complex $\mathbb{K}_{\leq n}$.

Step of Algorithm: find a vertex u for a possible retraction and remove u with all simplices containing it.

The algorithm terminates if there are no possible retractions.

As a result of such algorithm we obtain some vertex.

This is the leader election Algorithm L [7,8].

Algorithm 2.4 [obtaining a retractable complex]:

Input: a vertex u.

Step of Algorithm: add vertex v adjacent to some vertex u and all its neighbors to generate simplices containing $\{u, v\}$ of desired dimension.

The algorithm terminates after generating desired number of vertices.

As a result we obtain any retractable complex.

Similar algorithms were obtained in [9].

A complex $\mathbb{K}'$ is an extensor of a subcomplex $\mathbb{K}$, if a subcomplex $\mathbb{K}$ is a retract of $\mathbb{K}'$.

Fact 2.5. If a complex $\mathbb{K}'$ is an extensor of the retractable complex $\mathbb{K}$, then it is retractable.

From Theorem 2.1 we have:

Corollary 2.6. If a complex $\mathbb{K}'$ is an extensor of the retractable complex $\mathbb{K}$, then it has the fixed point property.

3 Collapsable complexes

The class of collapsable complexes is bigger than the class of retractable complexes.

Theorem 3.1. Every retractable complex is collapsable.

Proof. We show a construction of an elementary collapse. Let $\mathbb{K}_{\leq n}$ be a retractable complex. There are two vertices u, v and a retraction taking u to v and a subcomplex $\mathbb{K}'_{\leq n}$

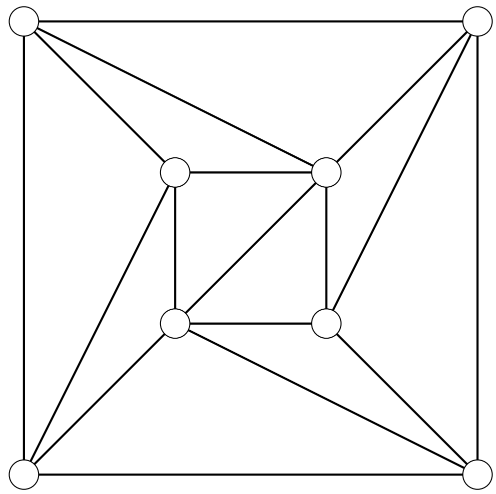

Figure 2 Collapsable complex $\mathbb{K}_{\leq 2}$(contains all possible 2-simplices on the picture) which is not retractable.

being a retract of $\mathbb{K}_{\leq n}$. Let us consider the set of all neighbors of u. From our assumption all those vertices are adjacent to v. Consider the star in $\mathbb{K}_{\leq n}$ with the center u. Let T be a maximal simplex of this star. There is also a simplex $S \in st_{\mathbb{K}}(u)$ which is a maximal proper subsimplex of T (not containing v) and thus S is a single simplex. We can define a sequence of elementary collapses of $\mathbb{K}_{\leq n}$ to obtain a complex $\mathbb{K}'_{\leq n}$. □

However, the converse of Theorem 3.1. is not true (see Figure 2).

In the complex $\mathbb{K}_{\leq 2}$ shown in the Figure 2, there are no possible retractions. Let us consider any pair $\{u, v\}$ of adjacent vertices of $\mathbb{K}_{\leq 2}$. Notice that for any choice of $\{u, v\}$ there exists a vertex x such that x, u are adjacent and x, v are not adjacent. If $r(u) = v$, there appears a new 1-simplex $\{x, v\}$. Thus obtained complex is not a subcomplex of $\mathbb{K}_{\leq 2}$ and the map r is not well defined retraction.

In fact, the proof of Theorem 3.1 defines the leader election algorithm [7] for collapsable complexes:

Algorithm 3.2 [reducing a collapsable complex to a ≤1-complex]:

Input: a collapsable complex $\mathbb{K}_{\leq n}$.

Step of Algorithm: find a single simplex $S \subset T$ (where T is the unique simplex in $\mathbb{K}_{\leq n}$) of the highest possible dimension (greater than 0), remove S and T.

The algorithm terminates if every single simplex is 0-simplex (a vertex).

As a result we obtain a spanning tree of $\mathbb{K}_{\leq n}$.

Algorithm 3.3 [reducing a retractable ≤1-complex (a tree) to its vertex]:

Input: a retractable ≤1-complex $\mathbb{L}_{\leq n}$, a vertex x of $\mathbb{L}_{\leq n}$.

Step of Algorithm: find a single 0-simplex $y \neq x$, remove it and the 1-complex containing it.

The algorithm terminates if there are no vertices but x.

As a result, we may obtain any arbitrarily chosen vertex of $\mathbb{L}_{\leq n}$.

4 Complexes without infinite paths

In this paragraph, we generalize the theorem of Rival and Nowakowski:

Theorem 4.1 ([10], Theorem 3). Let G be a graph with loops. Every edge-preserving map of set of $V(G)$ to itself fixes an edge if and only if (i) G is connected, (ii) G contains no cycles, and (iii) G contains no infinite paths.

We prove the fixed simplex property for the complexes which are not necessarily finite.

By an ∞-complex $\mathbb{K}_\infty$ defined on a set V we understand a family consisting of some n-simplices of $\mathbb{P}(V)$ with the property that for any n-simplex $S \in \mathbb{K}_\infty$, $\overline{S} \subset \mathbb{K}_\infty$; ($n \in N$).

An infinite path in a complex $\mathbb{K}_\infty$ is a sequence of vertices $\{s_0, s_1, \dots\}$ of $\mathbb{K}_\infty$ such that $\{s_i, s_{i+1}\}$ is 1-simplex of $\mathbb{K}_\infty (i \in N)$.

In case $s_k = s_{k+i}$ for some $k \in N$ and every $i \in N$ we define a finite path of the length k and we denote it by $P = \{s_0, s_1, \dots, s_k\}$. The length k of P we denote by $l(P)$.

Remark 4.2. An $\leq$1-complex consisting of vertices of some finite path $\{s_0, \dots, s_k\}$ and 1-complexes $\{s_i, s_{i+1}\}$ ($i \in \{0, 1, \dots, k - 1\}$) in case $s_i \neq s_j$ for $i \neq j$ ($i, j \in I_k$) is a retractable complex. So, it has the fixed simplex property.

A cycle is a finite path $\{s_0, s_1, \dots, s_k\}$ ($k \in N$) such that $\{s_0, s_k\} \in \mathbb{K}_\infty$.

A complex $\mathbb{K}_\infty$ is connected if every pair of vertices belongs to a finite path in $\mathbb{K}_\infty$.

Theorem 4.3. A connected complex $\mathbb{K}_\infty$ without infinite paths and with the property that every complex induced by a cycle is a retractable complex has the fixed simplex property.

Proof. Assume $\mathbb{K}_\infty$ is a complex containing no infinite paths. Suppose $f : V(\mathbb{K}_\infty) \to V(\mathbb{K}_\infty)$ is a simplicial map with no fixed simplex. Let us choose a vertex s_0 in $\mathbb{K}_\infty$ such that a path $P = \{s_0, \dots, f(s_0)\}$ has minimal length. Of course P contains at least two distinct vertices. Define $f^i(P) := \{f^i(s_0), \dots, f^{i+1}(s_0)\}$ ($i \geq 0$, $f^0(P) := P$). Because the length of P is minimal, then $l(f^i(P)) = l(f^{i+1}(P))$, $i \geq 0$. Without loss of generality, we may assume that $f^i(P) \cap f^{i+k}(P) = \varnothing$ for $k > 1$, $i \in N$. Otherwise $\mathbb{K}_\infty$ would contain a cycle and because it generates a retractable complex, so it would have the fixed simplex property by Theorem 2.1. Observe also that $f^i(P) \cap f^{i+1}(P) = \{f^{i+1}(s_0)\}$ for $i \geq 0$. Otherwise $f^i(P) = f^{i+1}(P)$ for some $i \geq 0$ and by Remark 4.2 there is a fixed simplex for f. Therefore, the complex $\mathbb{K}_\infty$ contains the infinite path $\{P, f(P), f^2(P), \dots\}$ and this contradicts our assumption. □

5 Recursively contractible complexes

A complex is recursively contractible if it is generated by an n-simplex (a simple complex [11]) or it is the union of two recursively contractible complexes such that their intersection is also a recursively contractible complex.

A complex is s-recursively contractible (tree like) if it is generated by an n-simplex or it is the union of two s-recursively contractible complexes such that their intersection is a complex generated by a simplex.

We showed that the s-recursively contractible complexes are a proper subclass of the retractable complexes:

Theorem 5.1. [5] For an s-recursively contractible complex $\mathbb{K}_{\leq n}$ we can obtain the complex generated by any simplex of $\mathbb{K}_{\leq n}$ by a sequence of retractions.

Corollary 5.2. [5] Every s-recursively contractible complex is retractable.

The converse of Corollary 5.2. is obviously not true (see Figure 3). Now, we show that the class of collapsable complexes is strictly contained in the class of *-recursively contractible complexes.

A complex is *-recursively contractible if it is generated by an n-simplex or it is the union of two *-recursively contractible complexes such that their intersection is a star.

Theorem 5.3. If an $\leq n$-complex $\mathbb{K}$ is collapsable, then it is *-recursively contractible.

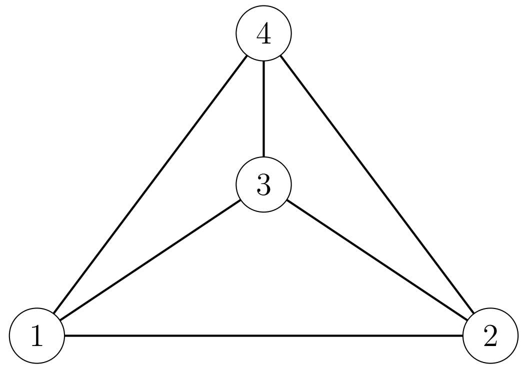

Figure 3 A retractable complex $\mathbb{K}_{\leq 2}$ (containing three 2-simplices: {123}, {134}, {234}) which is not *s*-recursively contractible.

Proof. Observe that a star of a vertex of a complex $\mathbb{K}$ is a collapsable complex and it is also recursively contractible.

If a complex $\mathbb{K}_{\leq n}$ is collapsable, then there exists a sequence of complexes (and elementary collapses) $\mathbb{L}^{k+1}, \mathbb{L}^{k}, \dots, \mathbb{L}^{0}$; $\mathbb{K}_{\leq_n} = \mathbb{L}^{k+1}$ and $\mathbb{L}^0$ is a 0-simplex ($k \in N$). For any complex $\mathbb{L}^{m+1}(m \in I_k)$ there is a single i-simplex S in $\mathbb{L}^{m+1}$ and a unique $(i+1)$-simplex T such that $S \subset T$, for some $i \in I_{n-1}$. Thus $\mathbb{L}^{m+1}$ is the union of complexes $\mathbb{L}^m$ and $\mathbb{K}_{\leq i+1}(T)$ and their intersection is a complex $\mathbb{K}_{\leq i+1}(T)\backslash\{S, T\}$ which is a star of a vertex. The complexes $\mathbb{K}_{\leq i+1}(T)$ and $\mathbb{K}_{\leq i+1}(T)\backslash\{S, T\}$ are *-recursively contractible ($i \in N$). The complex $\mathbb{L}^m$ can be represented as a union of a *-recursively contractible complex and the complex $\mathbb{L}^{m-1}$ and their intersection is a *-recursively contractible complex. Because the sequence of elementary collapses in complexes $\mathbb{L}^{m+1}$, $m \in I_k$ is finite and $\mathbb{L}^0$ is *-recursively contractible as a 0-simplex, then the complex $\mathbb{K}_{\leq n}$ is *-recursively contractible. □

There are some *-recursively complexes which are not collapsable (see Figure 4).

The ≤2-complex in the Figure 4 contains eight 2-simplices: {126}, {146}, {256}, {456}, {145}, {134}, {135}, {235}. The only single 1-simplices are: {12}, {23}, {34}. We need four

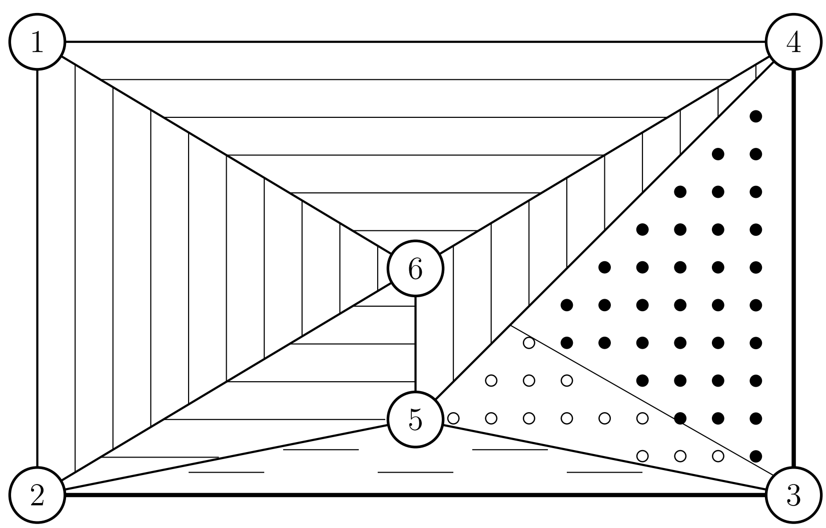

Figure 4 A construction of *-recursively contractible complex which is not collapsable.

copies of this complex taken in pairs for each we glue the thick 1-simplices {23}, {34} to obtain two collapsable complexes. Each of them has two single 1-simplices {12} and its copy) with common vertex: a star. We glue them again along these stars. The intersection is a star and the complex obtained is *-recursively contractible but not collapsable.

We know that a collapsable complex can be collapsed to any vertex. We may proceed collapsing beginning with maximal single i-simplices to obtain a tree. Thus it is collapsable to any chosen vertex. Collapsable complexes cannot be reduced by a sequence of elementary collapses to an arbitrarily chosen subcomplex. We construct a collapsable complex with only one single 1-simplex (see Figure 5).

We construct a complex as the union of two copies of the ≤2-complex presented on the Figure 5. In this case the copies differ by one vertex (the first copy has six vertices, the other has seven vertices: we add the vertex I here and, respectively, triangulate the simplex {126} onto {I16} and {I26}, adding the 1-simplex {I6}). We identify respective pairs of vertices 2, 3, 4, and the vertex 1 from first copy with the vertex I from the other copy. The obtained complex is still collapsable but has only one single 1-simplex {1I}.

Any *-recursively contractible complex is obviously recursively contractible but these classes are not equivalent (see Figure 6).

Consider the complex as a union of the following complexes. The first one consists of five vertices and edges as shown in the Figure 6 and the faces {124}, {134}, {135}, {145}, {235}, {245}. The second one is a copy of the first one but with three more vertices (A, B, C), 1-simplices {A2}, {AB}, {B5}, {C5}, {AC}, {A3}, {A5} and appropriate 2-simplices. Both complexes are collapsible, their intersection is a ≤2-complex {{1}, {2}, {3}, {4}, {12}, {23}, {34}} which is obviously collapsible, but the union does not have any single i-simplex (see Figure 4). Moreover, the obtained complex is not *-recursively contractible which can be verified by analyzing all its stars (removing any star of this complex does not disconnect it).

A graph G which generates a retractable complex $\mathbb{K}_G$ is called a retractable graph.

A graph G is triangulated if every cycle of length greater than 3 possesses a chord, i. e., an edge joining two nonconsecutive vertices of the cycle.

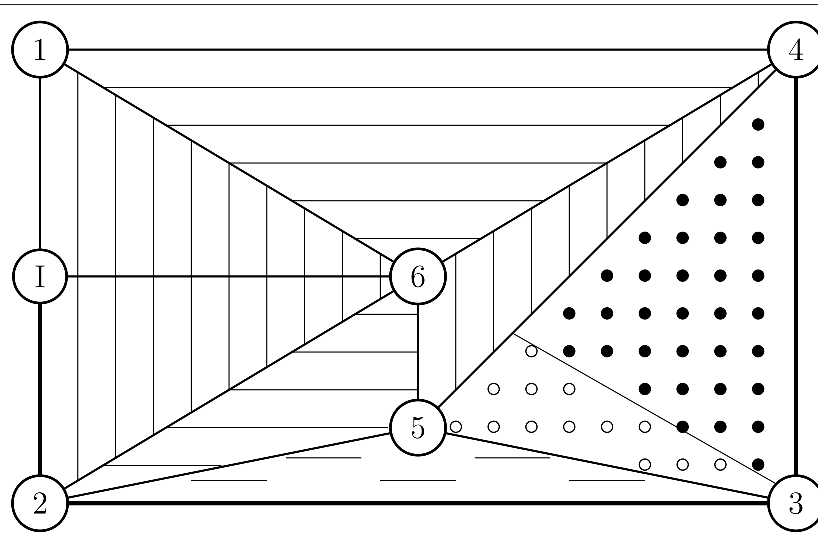

Figure 5 Collapsable complex with the only one single 1-simplex.

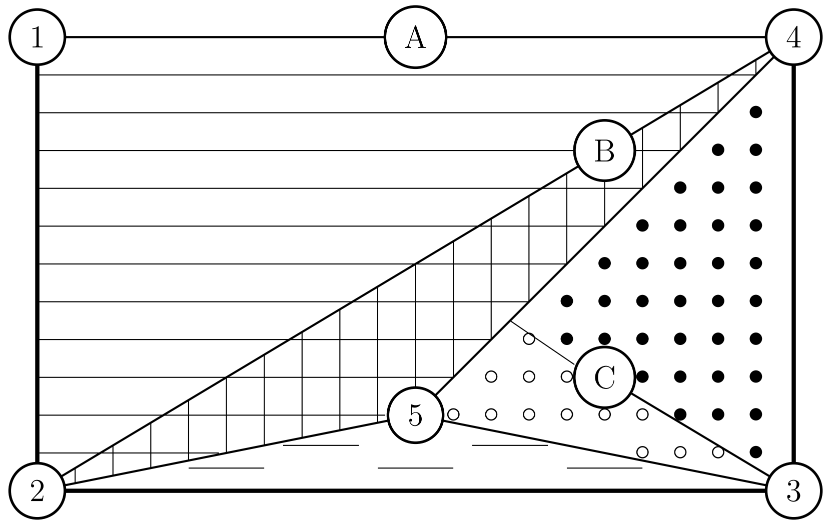

Figure 6 Recursively contractible complex which is not *-recursively contractible.

A clique in a graph G is a subgraph H of G with $V(H) \subset V(G)$, $\mathbb{E}(H) \subset \mathbb{E}(G)$ such that $\mathbb{E}(H) = \mathbb{P}_{\leq 1}(V(H))$.

A vertex x is called perfect if the set of its neighbors induces a clique.

Every triangulated graph G has a perfect elimination scheme (p. e. s.), i.e., we can always find a perfect vertex v in G and eliminate it with all edges e of G such that $v \in e$ (e.g., [[7], Theorem 1.1]).

A subset $S \subset V(G)$ is a vertex separator for nonadjacent vertices a, b if the removal of S from the graph G separates a and b into distinct connected subgraphs of G.

$S \subset V(G)$ is a minimal vertex separator for nonadjacent vertices a and b, if it is a vertex separator not properly containing any other vertex separator for a, b.

Observe that every (induced) subgraph of a triangulated graph is triangulated. Consider a complex generated by any triangulated graph (by covering by maximal cliques). It is an s-recursively contractible complex by

Fact 5.4. [12] A graph G is triangulated if and only if every minimal vertex separator induces a clique in G.

Let the vertices of a graph G be covered by its maximal cliques (the covering is unique). These cliques generate maximal simplices. The graph G is identified with a graph complex $\mathbb{K}_G$ consisting of these simplices and its subsimplices. There is one to one correspondence between the graph G and the graph complex $\mathbb{K}_G$ defined in that way.

Fact 5.5. Every triangulated graph generates an s-recursively contractible complex.

Any s-recursively contractible complex can be reduced, by a sequence of retractions, to an arbitrarily chosen subcomplex generated by some simplex. However for collapsable complexes, as well as for *-recursively contractible complexes, such reduction is not always possible (see Figure 2).

Author details
[1]Institute of Mathematics, Jan Kochanowski University, Kielce, Poland [2]Institute of Computer Science, Polish Academy of Sciences, Warsaw, Poland [3]Faculty of Mathematics and Information Science, Warsaw University of Technology, Warsaw, Poland

Authors' contributions

AI conceived of the study, participated in its design and coordination. AZ carried out research and drafted the manuscript. Both authors read and approved the final manuscript.

Competing interests

This research was partially supported by the National Science Centre, Poland (grant 6114/B/H03/2011/40) and by the Jan Kochanowski University in Kielce (grant BS 612439).

Received: 10 June 2011 Accepted: 8 February 2012 Published: 8 February 2012

References

1. Kaczynski, T, Mischaikow, K, Mrozek, M: Computational Homology. Springer-Verlag, New York, Inc (2004)
2. Whitehead, JHC: Simplicial spaces, nuclei and *m*-groups. Proc Lond Math Soc. **45**, 243–327 (1939). doi:10.1112/plms/s2-45.1.243
3. Wegner, G: d-Collapsing and Nerves of Families of Convex Sets. Archiv der Math-ematik. **26**, 317–321 (1975). doi:10.1007/BF01229745
4. Dey, TK, Edelsbrunner, H, Guha, S: Computational Topology. Contemp Math. **223**, 109–143 (1999)
5. Idzik, A, Zapart, A: Fixed Simplex Property for Retractable Complexes. Fixed Point Theory Appl **2010**, 7 (2010). Article ID 303640, doi:10.1155/2010/303640
6. Hell, P, Nešetřil, J: Graphs and Homomorphisms (Oxford Lecture Series in Mathematics and Its Applications). Oxford University Press Inc, New York (2004)
7. Mazurkiewicz, A: Local properties of triangular graphs. Fundamenta Informaticae. **79**, 487–495 (2007)
8. Mazurkiewicz, A: Local computations on triangular graphs. Fundamenta Infor-maticae. **100**, 117–140 (2010)
9. Mazurkiewicz, A: Locally derivable graphs. Fundamenta Informaticae. **75**, 335–355 (2007)
10. Nowakowski, R, Rival, I: Fixed-edge theorem for graphs with loops. J Graph Theory. **3**, 339–350 (1979). doi:10.1002/jgt.3190030404
11. Wieczorek, A: The Kakutani property and the fixed point property of topological spaces with abstract convexity. J Math Anal Appl. **168**, 483–499 (1992). doi:10.1016/0022-247X(92)90174-C
12. Golumbic, MC: Algorithmic Graph Theory and Perfect Graphs. In Annals of Discrete Mathematics, vol. 57,Elsevier (2004)

doi:10.1186/1687-1812-2012-11
Cite this article as: Idzik and Zapart: **Local properties of simplicial complexes.** *Fixed Point Theory and Applications* 2012 **2012**:11.